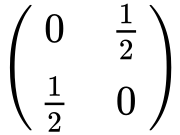
# 1.14 Introduction to matrices\_P\_2

**1a.** *[2 marks]*

## Markscheme

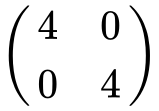
\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

***A*** =      ***A2  N2***

***[2 marks]***

**1b.** *[2 marks]*

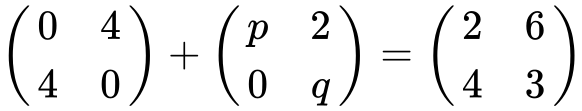
## Markscheme

***A*** =      ***A2  N2***

***[2 marks]***

**1c.** *[3 marks]*

## Markscheme

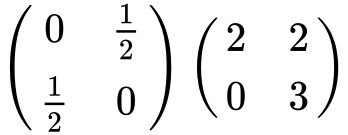
    ***(M1)***

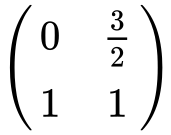
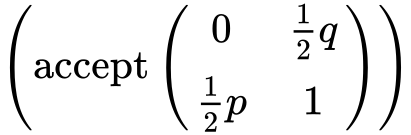
 = 2,  = 3   ***A1A1   N3***

**1d.** *[2 marks]*

## Markscheme

Evidence of attempt to multiply     ***(M1)***

*eg* ***AB*** = 

***AB*** =               ***A1  N2***

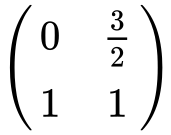
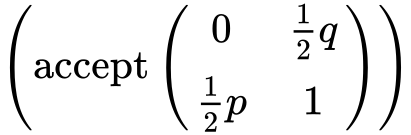
***[2 marks]***

**1e.** *[2 marks]*

## Markscheme

Evidence of correct approach    ***(M1)***

*eg* ***X*** = ***AB***, setting up a system of equations

***X*** =               ***A1  N2***

***[2 marks]***

**2a.** *[1 mark]*

## Markscheme

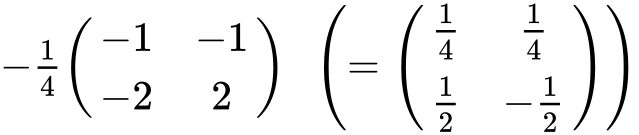
\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

det ***M*** = −4       ***A1  N1***

***[1 mark]***

**2b.** *[2 marks]*

## Markscheme

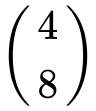
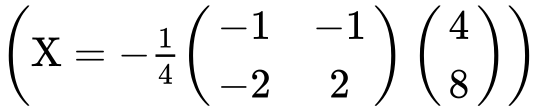
***M*** =       ***A1A1 N2***

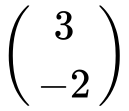
**Note:**Award ***A1***for  and ***A1***for the correct matrix.

***[2 marks]***

**2c.** *[3 marks]*

## Markscheme

***X*** = ***M***         ***M1***

***X***=      (, )      ***A1A1   N0***

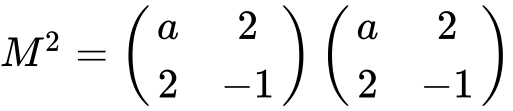
**Note:**Award no marks for an **algebraic** solution of the system , .

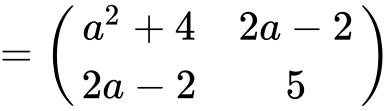
***[3 marks]***

**3a.** *[4 marks]*

## Markscheme

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.



       ***(A1)(A1)(A1)(A1)***

***[4 marks]***

**3b.** *[2 marks]*

## Markscheme

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.



      ***(A1)***

Substituting:       ***(A1)***

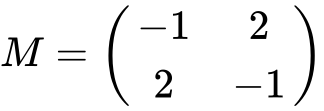
**Note:** Candidates may solve  to give , and then show that only  satisfies .

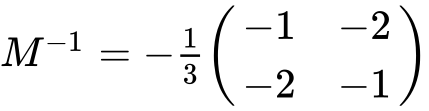
***[2 marks]***

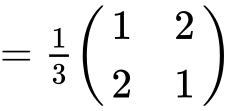
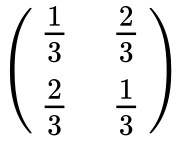
**3c.** *[6 marks]*

## Markscheme

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

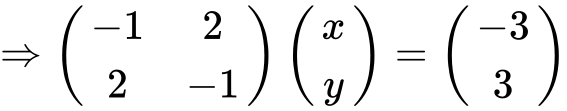


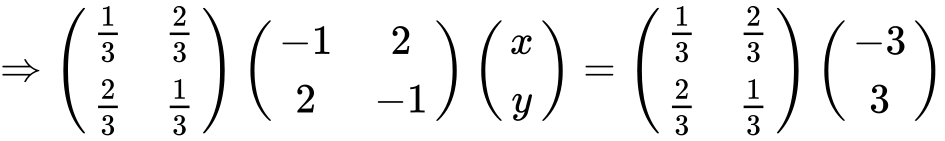
       ***(M1)***

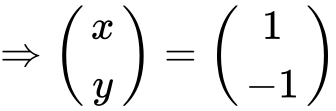
 or       ***(A1)***





       ***(M1)(M1)***

      ***(A1)***

       ***(A1)***

ie   



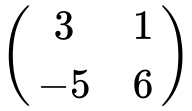
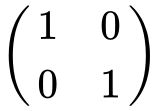
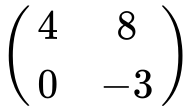
**Note:** The solution must use matrices. Award no marks for solutions using other methods.

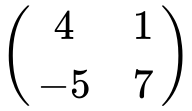
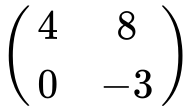
***[6 marks]***

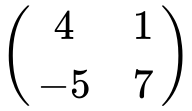
**4.** *[8 marks]*

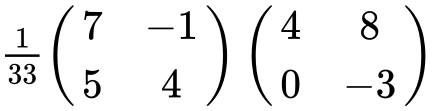
## Markscheme

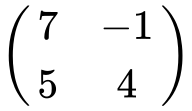
\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

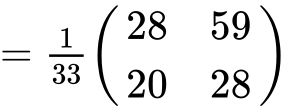
***X*** + ***X*** = 

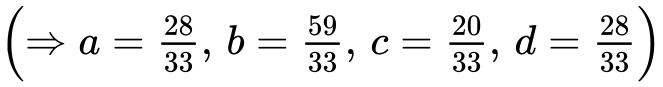
***X*** =         ***(M1)***

Pre-multiply by inverse of         ***(M1)***

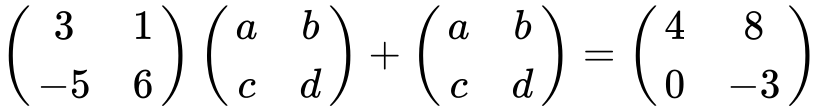
***X*** =         ***(A1)(A1)***

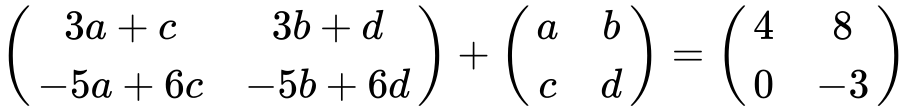
**Note:**Award ***(A1)*** for determinant, ***(A1)*** for matrix .

        ***(A1)(A1)(A1)(A1)***



**OR**

        ***(A1)***

        ***(A1)***

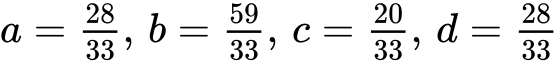


        ***(A1)***



        ***(A1)***

**Notes:** Award ***(A1)*** for each pair of equations.   
Allow **ft** from their equations.

        ***(A1)(A1)(A1)(A1)***

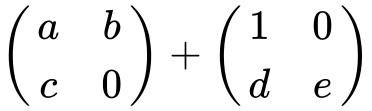
**Note:**Award ***(A0)(A0)(A1)(A1)*** if the final answers are given as decimals *ie* 0.848, 1.79, 0.606, 0.848.

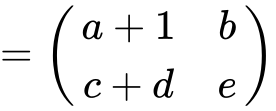
***[8 marks]***

**5a.** *[2 marks]*

## Markscheme

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

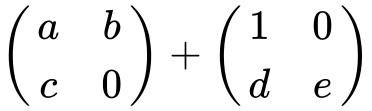
***A*** + ***B*** = 

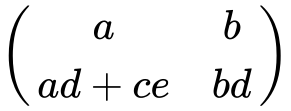
            ***A2***

***[2 marks]***

**5b.** *[4 marks]*

## Markscheme

***AB*** =             ***A1A1A1A1***

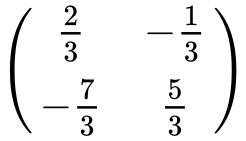
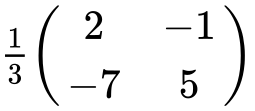
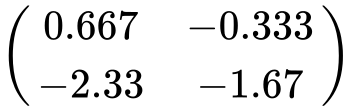
**Note**: Award ***N2*** for finding **BA** = .

***[4 marks]***

**6a.** *[2 marks]*

## Markscheme

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

***A*** =  or  or          ***(A1)(A1)(N2)***

***[2 marks]***

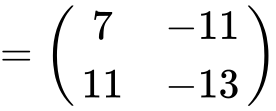
**6b.** *[3 marks]*

## Markscheme

***AX*** = ***C*** − ***B*** (may be implied)        ***(A1)***

***X*** = ***A*(*C****−****B*)**        ***(A1)***

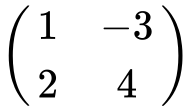
***D*** = ***C*** *−****B***

        ***(A1)  (N3)***

***[3 marks]***

**6c.** *[2 marks]*

## Markscheme

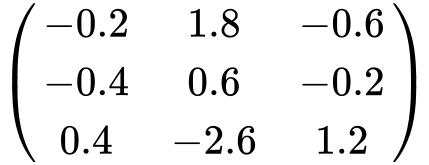
***X*** =        ***(A2)  (N2)***

***[2 marks]***

**7a.** *[2 marks]*

## Markscheme

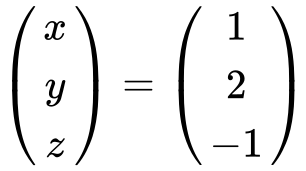
\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

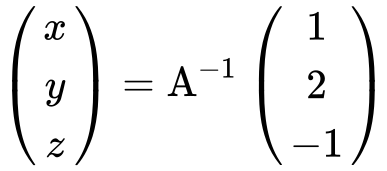
***A*** =             ***A2    N2***

***[2 marks]***

**7b.** *[4 marks]*

## Markscheme

For recognizing that the equations may be written as ***A***      ***(M1)***

For attempting to calculate        ***(M1)***

 = 4,   = 1,   = −6       ***A2    N4***

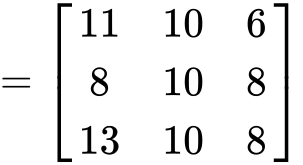
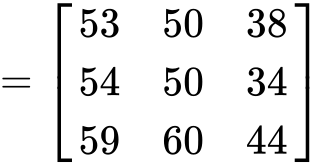
***[4 marks]***

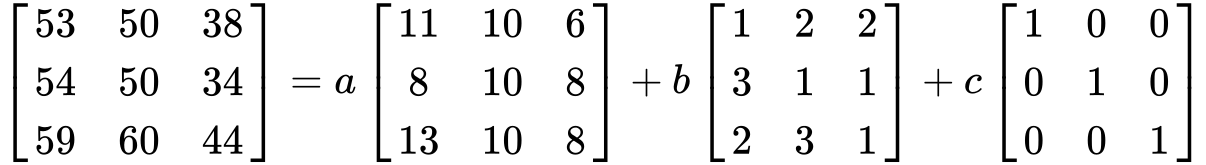
**8a.** *[7 marks]*

## Markscheme

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

**METHOD 1**

***M*** ; ***M***      ***(A1)(A1)***

let        ***(M1)***

then, for example

11*a* + *b* + *c* = 53

10*a* + 2*b* = 50        ***M1A1***

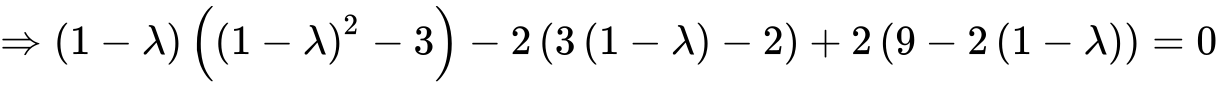
6*a* + 2*b* = 38

the solution is *a* = 3, *b* =10, *c* =10      ***(M1)A1***

(***M*** = 3***M*** +10***M*** +10***I***)

**METHOD 2**

det(***M*** − ***I***) = 0       ***(M1)***

       ***M1A1***

       ***M1A1***

applying the Cayley – Hamilton theorem       ***(M1)***

***M*** = 3***M*** +10***M*** +10***I*** and so *a* = 3, *b* =10, *c* =10       ***A1***

***[7 marks]***

**8b.** *[2 marks]*

## Markscheme

***M*** = 3***M*** + 10***M*** + 10***M     M1***

= 3(3***M*** + 10***M*** + 10***I***) + 10***M*** + 10***M      M1***

=19***M*** + 40***M*** +30***I       AG***

***[2 marks]***

**8c.** *[6 marks]*

## Markscheme

the statement is true for *n* = 3 as shown in part (a)       ***A1***

assume true for *n* = *k*, *ie* ***M*** = *p****M*** + *q****M*** + *r****I***       ***M1***

**Note:** Subsequent marks after this ***M1***are independent and can be awarded.

***M*** = *p****M*** + *q****M***  + *r****M*** ***M1***

= *p*(3***M*** +10***M*** +10***I***) + *q****M*** + *r****M         M1***

= (3*p* +*q*)***M*** + (10*p* + *r*)M + 10*p****I***     ***A1***

hence true for *n* = *k* ⇒ true for *n* = *k* +1 and since true for *n* = 3, the statement is proved by induction       ***R1***

**Note:** Award ***R1*** provided at least four of the previous marks are gained.

***[6 marks]***

**8d.** *[2 marks]*

## Markscheme

***M*** = 3***M*** + 10***I*** + 10***M***     ***M1***

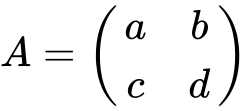
***M*** = (***M*** − 3***M*** − 10***I***)     ***A1***

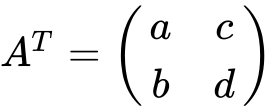
***[2 marks]***

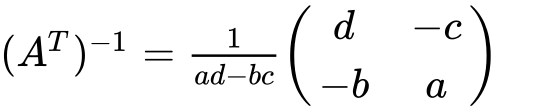
**9a.** *[8 marks]*

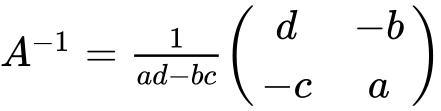
## Markscheme

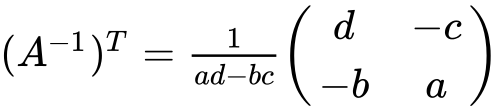
\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

(i)     

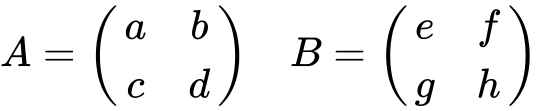
     ***M1***

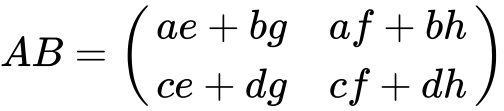
(which exists because )     ***A1***

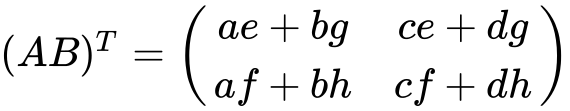
     ***M1***

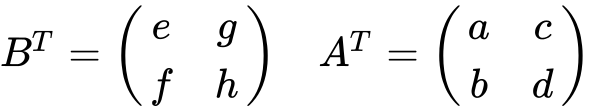
     ***A1***

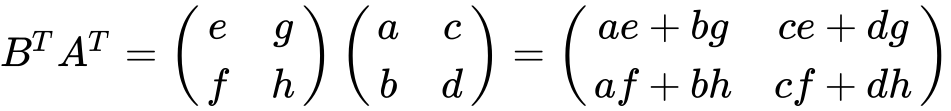
hence  as required ***AG***

(ii)     

     ***M1***

     ***A1***

     ***M1***

     ***A1***

hence      ***AG***

**9b.** *[8 marks]*

## Markscheme

 is reflexive since  and      ***A1***

     ***M1A1***

 from a (i)     ***A1***

which is of the correct form, hence symmetric     ***AG***

 and      ***M1***

**Note:**Allow use of  rather than  in this line.

     ***M1A1***

 from a (ii)     ***A1***

this is of the correct form, hence transitive

hence  is an equivalence relation     ***AG***

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